

# 2000 Advanced Placement Program® Free-Response Questions

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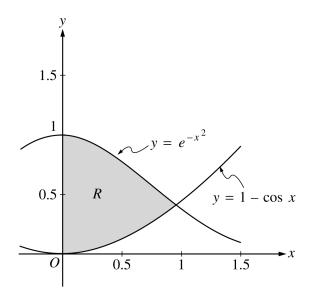


# CALCULUS AB SECTION II, Part A

Time—45 minutes

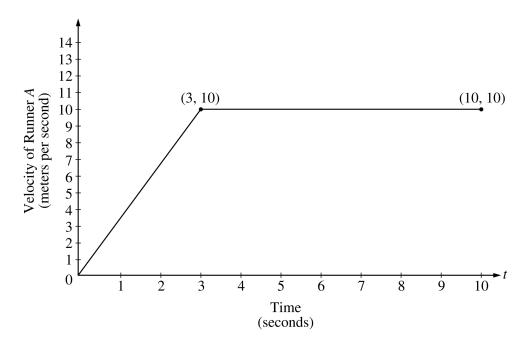
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

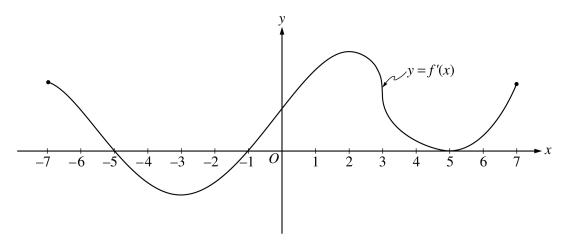


- 1. Let R be the shaded region in the first quadrant enclosed by the graphs of  $y = e^{-x^2}$ ,  $y = 1 \cos x$ , and the y-axis, as shown in the figure above.
  - (a) Find the area of the region R.
  - (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
  - (c) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of this solid.

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- 2. Two runners, A and B, run on a straight racetrack for  $0 \le t \le 10$  seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by  $v(t) = \frac{24t}{2t+3}$ .
  - (a) Find the velocity of Runner A and the velocity of Runner B at time t=2 seconds. Indicate units of measure.
  - (b) Find the acceleration of Runner A and the acceleration of Runner B at time t=2 seconds. Indicate units of measure.
  - (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval  $0 \le t \le 10$  seconds. Indicate units of measure.



- 3. The figure above shows the graph of f', the derivative of the function f, for  $-7 \le x \le 7$ . The graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3.
  - (a) Find all values of x, for -7 < x < 7, at which f attains a relative minimum. Justify your answer.
  - (b) Find all values of x, for -7 < x < 7, at which f attains a relative maximum. Justify your answer.
  - (c) Find all values of x, for -7 < x < 7, at which f''(x) < 0.
  - (d) At what value of x, for  $-7 \le x \le 7$ , does f attain its absolute maximum? Justify your answer.

#### **END OF PART A OF SECTION II**

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# **CALCULUS AB SECTION II, Part B**

Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

- 4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of  $\sqrt{t+1}$  gallons per minute, for  $0 \le t \le 120$  minutes. At time t=0, the tank contains 30 gallons of water.
  - (a) How many gallons of water leak out of the tank from time t = 0 to t = 3 minutes?
  - (b) How many gallons of water are in the tank at time t = 3 minutes?
  - (c) Write an expression for A(t), the total number of gallons of water in the tank at time t.
  - (d) At what time t, for  $0 \le t \le 120$ , is the amount of water in the tank a maximum? Justify your answer.
- 5. Consider the curve given by  $xy^2 x^3y = 6$ .
  - (a) Show that  $\frac{dy}{dx} = \frac{3x^2y y^2}{2xy x^3}$ .
  - (b) Find all points on the curve whose *x*-coordinate is 1, and write an equation for the tangent line at each of these points.
  - (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.
- 6. Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .
  - (a) Find a solution y = f(x) to the differential equation satisfying  $f(0) = \frac{1}{2}$ .
  - (b) Find the domain and range of the function f found in part (a).

#### **END OF EXAMINATION**

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